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1)

$$\begin{cases} 7x + y = 23 \\ -5x + 3y = 1 \end{cases}$$

$$\Delta = \begin{vmatrix} 7 & 1 \\ -5 & 3 \end{vmatrix} = 21 + 5 = 26 \neq 0,$$

$$\Delta_x = \begin{vmatrix} 23 & 1 \\ 1 & 3 \end{vmatrix} = 69 - 1 = 68$$

$$x = \frac{\Delta_x}{\Delta} = \frac{68}{26} = \frac{34}{13}$$

$$\Delta_y = \begin{vmatrix} 7 & 23 \\ -5 & 1 \end{vmatrix} = 7 + 115 = 122$$

$$y = \frac{\Delta_y}{\Delta} = \frac{122}{26} = \frac{61}{13}$$

$$\therefore x = \frac{34}{13}, y = \frac{61}{13}$$

$$1) 7 \cdot \frac{34}{13} + \frac{61}{13} = \frac{238}{13} + \frac{61}{13} = \frac{299}{13} = 23$$

$$2) -5 \cdot \frac{34}{13} + 3 \cdot \frac{61}{13} = -\frac{170}{13} + \frac{183}{13} = \frac{13}{13} = 1$$

$$\begin{cases} 2x - 3y + z = 2 \\ 2x + y - 4z = 9 \\ 6x - 5y + 2z = 17 \end{cases}$$

$$\begin{aligned} D &= \begin{vmatrix} 2 & -3 & 1 \\ 2 & 1 & -4 \\ 6 & -5 & 2 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & -4 \\ -5 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & -4 \\ 6 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 6 & -5 \end{vmatrix} = \\ &= 2 \cdot (2 - 20) + 3 \cdot (4 + 24) + (-10 - 6) = -36 + 84 - 16 = 32 \neq 0, \end{aligned}$$

$$\begin{aligned} D_x &= \begin{vmatrix} 2 & -3 & 1 \\ 9 & 1 & -4 \\ 17 & -5 & 2 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & -4 \\ -5 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} 9 & -4 \\ 17 & 2 \end{vmatrix} + \begin{vmatrix} 9 & 1 \\ 17 & -5 \end{vmatrix} = \\ &= 2 \cdot (2 - 20) + 3 \cdot (18 + 68) + (-45 - 17) = -36 + 258 - 62 = 160 \end{aligned}$$

$$x = \frac{D_x}{D} = \frac{160}{32} = 5$$

$$\begin{aligned} D_y &= \begin{vmatrix} 2 & 2 & 1 \\ 2 & 9 & -4 \\ 6 & 17 & 2 \end{vmatrix} = 2 \cdot \begin{vmatrix} 9 & -4 \\ 17 & 2 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & -4 \\ 6 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 9 \\ 6 & 17 \end{vmatrix} = \\ &= 2 \cdot (18 + 68) - 2 \cdot (4 + 24) + (34 - 54) = 172 - 56 - 20 = 96 \end{aligned}$$

$$y = \frac{D_y}{D} = \frac{96}{32} = 3$$

$$\begin{aligned} D_z &= \begin{vmatrix} 2 & -3 & 2 \\ 2 & 1 & 9 \\ 6 & -5 & 17 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & 9 \\ -5 & 17 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & 9 \\ 6 & 17 \end{vmatrix} + 2 \cdot \begin{vmatrix} 2 & 1 \\ 6 & -5 \end{vmatrix} = \\ &= 2 \cdot (17 + 45) + 3 \cdot (34 - 54) + 2 \cdot (-10 - 6) = 124 - 60 - 32 = 32 \end{aligned}$$

$$z = \frac{D_z}{D} = \frac{32}{32} = 1$$

$$\therefore x = 5, y = 3, z = 1.$$

$$\begin{cases} 2x - 3y + z = 2 \\ 2x + y - 4z = 9 \\ 6x - 5y + 2z = 17 \end{cases}$$

$$AX = b, \quad A = \begin{pmatrix} 2 & -3 & 1 \\ 2 & 1 & -4 \\ 6 & -5 & 2 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 2 \\ 9 \\ 17 \end{pmatrix}$$

$$X = A^{-1}b$$

$$A^{-1} = \frac{1}{|A|} \cdot A_*^T, \quad A_*^T -$$

A.

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & -3 & 1 \\ 2 & 1 & -4 \\ 6 & -5 & 2 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & -4 \\ -5 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & -4 \\ 6 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 6 & -5 \end{vmatrix} = \\ &= 2 \cdot (2 - 20) + 3 \cdot (4 + 24) + (-10 - 6) = -36 + 84 - 16 = 32 \end{aligned}$$

$$M = \begin{pmatrix} -18 & 28 & -16 \\ -1 & -2 & 8 \\ 11 & -10 & 8 \end{pmatrix} -$$

A.

$$A_* = \begin{pmatrix} -18 & -28 & -16 \\ 1 & -2 & -8 \\ 11 & 10 & 8 \end{pmatrix} -$$

$$A_*^T = \begin{pmatrix} -18 & 1 & 11 \\ -28 & -2 & 10 \\ -16 & -8 & 8 \end{pmatrix} -$$

$$A^{-1} = \frac{1}{32} \begin{pmatrix} -18 & 1 & 11 \\ -28 & -2 & 10 \\ -16 & -8 & 8 \end{pmatrix}$$

$$X = A^{-1}b = \frac{1}{32} \begin{pmatrix} -18 & 1 & 11 \\ -28 & -2 & 10 \\ -16 & -8 & 8 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 9 \\ 17 \end{pmatrix} = \frac{1}{32} \begin{pmatrix} -18 \cdot 2 + 1 \cdot 9 + 11 \cdot 17 \\ -28 \cdot 2 - 2 \cdot 9 + 10 \cdot 17 \\ -16 \cdot 2 - 8 \cdot 9 + 8 \cdot 17 \end{pmatrix} = \frac{1}{32} \begin{pmatrix} 160 \\ 96 \\ 32 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$$

$$: x = 5, y = 3, z = 1$$